

Finite Math - Spring 2019
Lecture Notes - 5/14/2019

HOMework

- Section 8.3 - 7, 8, 9, 10, 11, 12, 20, 21, 22, 43, 44, 23, 25, 27, 29, 31, 33, 45, 46, 49, 59, 60, 81, 83, 84

SECTION 8.3 - CONDITIONAL PROBABILITY,
INTERSECTION, AND INDEPENDENCE

Conditional Probability.

Example 1. *Suppose we have a pot with 5 red balls and 7 green balls inside. What is the probability that you will draw a red ball? a green ball?*

Example 2. *Suppose we drew a red ball out of the pot and set it aside. Now what is the probability to draw a red ball? a green ball?*

In the experiments we were doing earlier that involved doing things multiple times, the outcome of the first thing did not influence the later outcomes. As we can see with the previous example, this is not the case for this experiment (which is known as “sampling without replacement”). This is the notion of *conditional probability*, denoted by $P(A|B)$ “the probability of A given B .”

Definition 1 (Conditional Probability). *For events A and B in an arbitrary sample space S , we define the conditional probability of A given B by*

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(B) \neq 0$.

Example 3. *Suppose we roll a single fair die (6 sided), but cannot see the outcome. What is the probability that we rolled a prime number given that someone has told us that we rolled an odd number?*

Application.

Example 4. *Suppose that city records produced the following probability data on a driver being in an accident on the last day of a Memorial Day weekend:*

	Accident (A)	No Accident (A')	Totals
Rain (R)	.025	.335	.360
No Rain (R')	.015	.625	.640
Totals	.040	.960	1.000

- Find the probability of an accident, rain or no rain.*
- Find the probability of rain, accident or no accident.*
- Find the probability of an accident and rain.*
- Find the probability of an accident, given rain.*

Intersections of Events. In the previous section we came up with a rule for finding the probability of the union of events, but now we can finally come up with one for the intersection of events by using the definition of conditional probability. Given events A and B , if their probabilities are non zero, we can use the facts that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{and} \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

and solve for $P(A \cap B)$ in the first and $P(B \cap A)$ in the second to get

$$P(A \cap B) = P(A|B)P(B) \quad \text{and} \quad P(B \cap A) = P(B|A)P(A)$$

and since $A \cap B = B \cap A$, we get

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A).$$

We can use either product to compute the probability of the intersection.

Example 5. *If 40% of a department store's customers are male and 80% of the male customers have store credit cards, what is the probability that a customer selected at random is a male and has a store credit card?*

Example 6. *An automobile manufacturer produces 37% of its cars at plant A. If 5% of the cars manufactured at plant A have defective emission control devices, what is the probability that one of this manufacturer's cars was manufactured at plant A and has a defective emission control device?*

Probability Trees. We can create trees diagrams based on the number of outcomes in a sequence of experiments similar to how we used them for counting problems.

Example 7. *Two balls are drawn in succession, without replacement, from a pot containing 3 blue balls and 2 white balls. What is the probability of drawing a white ball on the second draw?*

Example 8. *Two balls are drawn in succession without replacement from a box containing 4 red and 2 white balls. What is the probability of drawing a red ball on the second draw?*

Example 9. *Consider again the experiment where two balls are drawn in succession, without replacement, from a pot containing 3 blue balls and 2 white balls. What difference would it make if we changed “without replacement” to “with replacement”? Let A be the event that a white ball is pulled on the second draw and let B be the event that a white ball is pulled on the first draw. Compute $P(A|B)$ and $P(A)$ in both scenarios: without and with replacement. What do you notice?*

Definition 2 (Independence). *If A and B are any events in a sample space S , we say that A and B are independent if*

$$P(A \cap B) = P(A)P(B).$$

Otherwise, A and B are called dependent events.

A consequence of this definition is that if A and B are independent events, we have

$$P(A|B) = P(A)$$

and

$$P(B|A) = P(B).$$

Example 10. *A single card is drawn from a standard 52-card deck. Test the following for independence:*

(a)

E = *the drawn card is a spade*

F = *the drawn card is a face card*

(b)

G = *the drawn card is a king*

H = *the drawn card is a queen*